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10/27/2007

Find the y-intercept of the normal to the parabola $y = x^2$ at the point (a,a^2), where a cannot equal zero. Find the value of this y-intercept as a approaches zero.

First, find the derivative of the parabola at x = a. This gives you the equation of the TANGENT line to the parabola.

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

$$\left.\frac{dy}{dx}\right|_{x=a} = 2a$$

Now note that the NORMAL line at point x =a has a slope that is the NEGATIVE RECIPROCAL of the TANGENT line at point x = a

Slope of tangent line = 2a

Slope of normal line = $-\frac{1}{2a}$

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Find the y-intercept of the normal to the parabola y = x^2 at the point (a,a^2), where a cannot equal zero. Find the value of this y-intercept as a approaches zero.

Now, we need to find the equation of the NORMAL line knowing that:
-Slope = -1/2a
-Point = (a,a^2)

$$(y - y_o) = m(x - x_o)$$

$$(y - a^2) = -\frac{1}{2a}(x - a)$$

$$(y - a^2) = -\frac{1}{2a}x + \frac{1}{2}$$

$$y = -\frac{1}{2a}x + \left(\frac{1}{2} + a^2\right)$$

This is the y-intercept

The value of the y-intercept as a approaches zero can be written as a limit.

$$\lim_{a \to 0}\left(\frac{1}{2} + a^2\right) = \left(\frac{1}{2} + 0\right) = \frac{1}{2}$$

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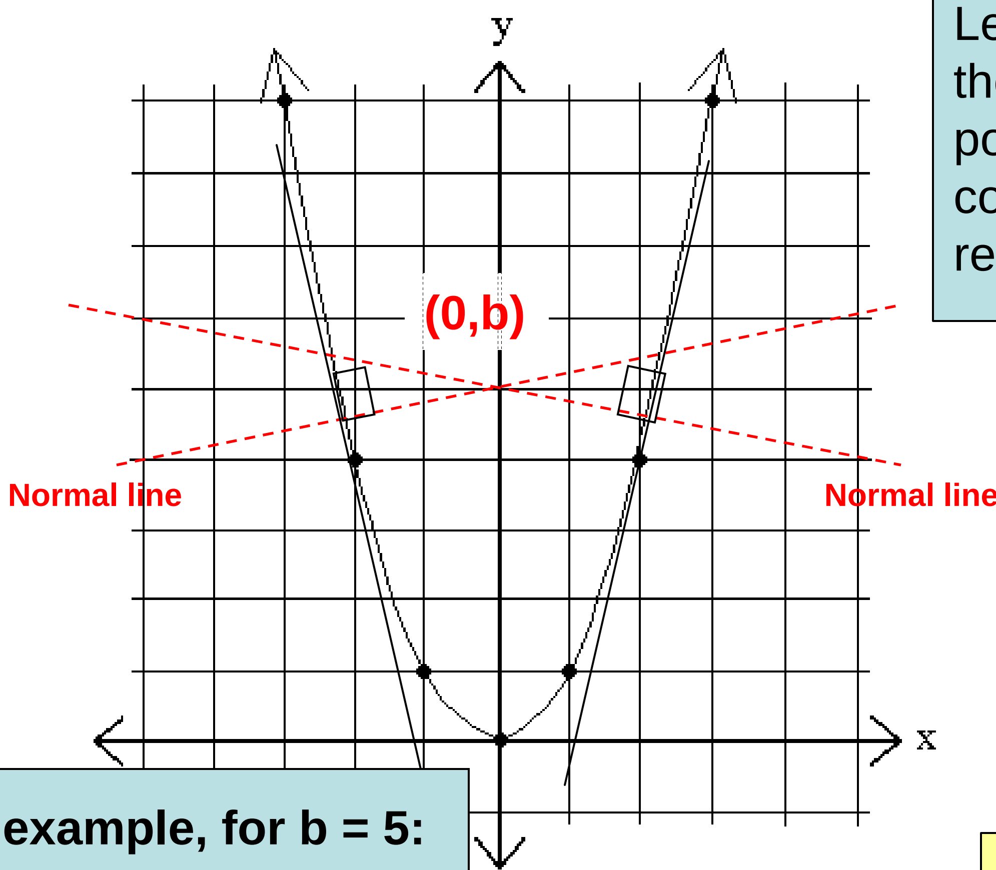
For each b greater than zero, determine the number of normals that can be drawn from the point (0,b) to the parabola x^2.



Let's use the previous equation for the NORMAL line and plug in the point (0,b) to see if we can conclude something about how b relates to a.

$$y = -\frac{1}{2a}x + \left(\frac{1}{2} + a^2\right)$$

$$b = -\frac{1}{2a}(0) + \left(\frac{1}{2} + a^2\right)$$

$$a = \pm\sqrt{b - \frac{1}{2}}$$

For each value of b, the normal line can only intersect the parabola at 2 values

For example, for b = 5:

$$a = -\frac{3}{\sqrt{2}}\ and\ +\frac{3}{\sqrt{2}}$$