Fluid Dynamics

Hydrostatics

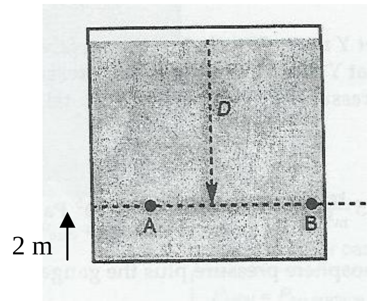
§ Hydrostatics is the study of fluids (hydro) at rest (statics)
- o Fluids are substances that flow. Liquids AND gases are fluids.

§ Density is the amount of mass contained in a unit of volume.
- o $r = \frac{mass}{volume}$
- o The density of water (which you should memorize) is 1000 kg/m^3 or 1 g/cm^3. (Note that 1 mL = 1 cm^3)
- o Specific gravity is a unitless number that tells you how dense something is **compared to density of water**.
  - § $\text{specific gravity} = \frac{r}{r_{H_2O}}$
  - § For example, ethanol has a specific gravity of 0.8. This means that its density is really 800 kg/m^3 or 0.8 g/cm^3.
- o The density of a solid and a liquid does not change much with surrounding temperature and pressure. However, the density of a gas can vary widely according to the Ideal Gas Law: $r_{gas} = \frac{mass}{volume} = \frac{mP}{nRT}$.

§ Pressure tells you how much contact force (perpendicular to surface) is exerted on an object's surface area within a fluid.
- o The formula for pressure is: $P = \frac{force_{\perp}}{area}$ (force **perpendicular** to the area)
- o Pressure is a **scalar**.
- o Pressure is measured in N/m^2 or Pascals (Pa). Note from general chemistry that pressure is also measured in atm or mm Hg. To get a sense of how these units are related, the atmospheric pressure at sea level is 1 atm = 100 kPa = 760 mm Hg.
- o The pressure of an object in a fluid (or **hydrostatic gauge pressure**) surprisingly does **not depend on its surface area** but the depth of the object and the density of the fluid: $P_{gauge} = r_{fluid} gD$ (where D is **depth below surface of the fluid**)

The MCAT will trick you on this by given you not the depth of an object but the height of an object above the bottom of a container. If ethanol fills the container to a height of 8 m, what is the pressure at a point 2 m from the bottom of the container? The depth is 6 m, **NOT** 2 m.

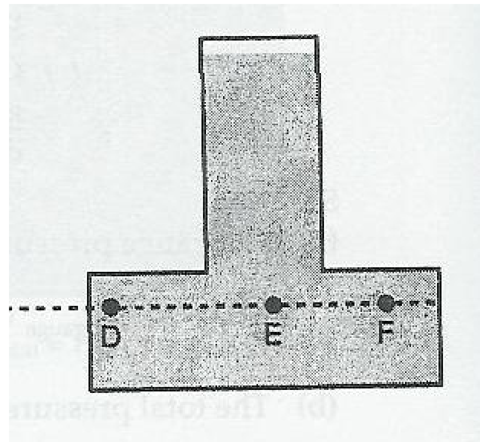
$$P_{gauge} = r_{fluid} gD = (800kg/m^3)(10m/s^2)(6m) = 48kPa$$



- If a fluid (that isn't the air) is **open to the air or atmosphere**, there is an **atmospheric pressure** you have to take into account when finding the total pressure exerted on an object inside a fluid.
- This means that the total pressure is: $P_{total} = P_{atm} + P_{gauge}$

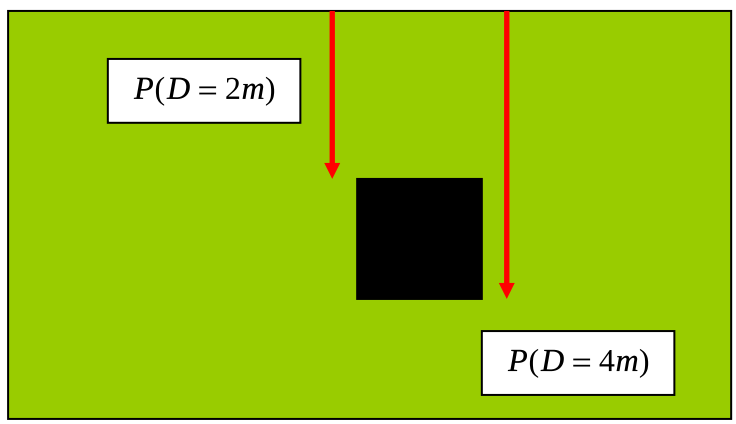
Exercise:

§ Three objects are submerged in a fluid in a closer container. These objects are located at points D, E and F. Which of the following statements is true?



A. The object at E experiences a greater gauge pressure than the objects at D. This is false because gauge pressure is the same at equal depths REGARDLESS of the shape of the container.
B. The object at D experiences a greater gauge pressure than the object at E. See above. The gauge pressure is equal at D, E and F.
C. If the container was open, the object at F would experience a greater total pressure. This is true because opening the container adds atmospheric pressure to the total pressure at F (and E and D as well).
D. If the density of fluid was decreased, all the objects will experience a greater total pressure. This is false because increasing density should increase hydrostatic pressure, and hence, total pressure as well.

§ Buoyancy is the ability to float in a fluid. Objects float because of buoyancy force. Where does buoyancy force comes from?



Buoyancy force comes from the difference between the pressure on the top surface of an object and the pressure on the bottom surface of an object. (Note that the pressures on the sides of the objects just cancel out.)

§ Archimedes' Principle tells us how to calculate the buoyant force.

- $F_b = r_{fluid} g V_{submerged}$
- Intuitively, the strength of the buoyant force is equal to the weight of the displaced fluid. For example, that if you step into a full bathtub, you will displace a volume of water equal to your weight.
- Note that buoyant force increases the more an object is submerged in a fluid. In fact, the maximum buoyant force occurs when the object is completely submerged.

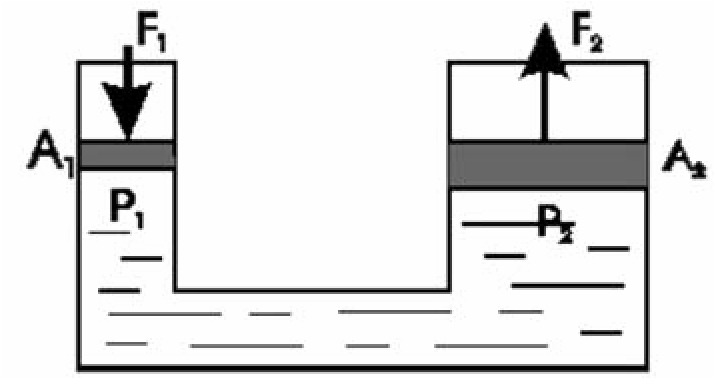
§ Why don't all objects float? Objects float only if their density is **LESS** than the density of the fluid. If an object has a **greater density** than the fluid, it will **ALWAYS SINK**. (Note that an object that is equal to the density of the fluid will stay in place—neither float nor sink.)

- The MCAT loves to ask about the density of an object that is submerged in a fluid by a certain fraction. They usually trick you by telling you how much an object's volume is **above** the fluid (or unsubmerged).
  - $\frac{r_{object}}{r_{fluid}} = \frac{V_{submerged}}{V_{total}} = \%\ \text{Volume submerged}$
  - Remember that an object that floats has a buoyant force equal to its own weight.

Example: If 20% of the volume of an object is above the surface of the fluid, then the density of the object is what percent of the density of the surrounding fluid?

If 20% of the volume of an object is above the surface of the fluid, THEN 80% of the object's volume is SUBMERGED. According to the equation: $\frac{r_{object}}{r_{fluid}} = \frac{V_{submerged}}{V_{total}} = 80\%$. Hence, 80% is the percent of the object's density compared to the density of the surrounding fluid (Note that if the surrounding fluid is water, we would really be asking for the specific gravity).

- Pascal's law tells us that a confined fluid will transmit an externally applied pressure to all parts of the fluid.
  - In the diagram, the pressure at location 1 is equal to the pressure at location 2.
  - If this is the case, then:



$$P_1 = \frac{F_1}{A_1}$$

$$P_2 = \frac{F_2}{A_2}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

  - This means that if apply a certain force on a small cross-sectional area, then you get a larger output force where there is a larger cross-sectional area.
  - What is the catch, since you get a larger output force than input force? Well, you can't cheat work. The work you apply at one end is the same work you get on the other end. This means that if you displace the fluid by a volume, then you push the same volume of fluid on the other end.
  - What is the hidden assumption in this problem? (Will Pascal's law apply for air?) The hidden assumption is that the fluid must be INCOMPRESSIBLE or constant density. If the fluid was compressible (like air), then displacing a fluid on one end by a certain volume will displace the fluid on the other end by a LESSER volume (because volume is compressed).

Exercises:

§ An object of mass 2 kg floats motionless in a fluid of specific gravity 0.8. What is the magnitude of the buoyant force? (Use g = 10 m/s^2)

A. 8 N
B. 16 N
C. 20 N
D. 25 N

If an object is floating, then it is not accelerating and buoyant force is precisely equal to weight. In this case, the buoyant force is equal to $F_B = W = (2kg)(10m/s^2) = 20N$. Note that the information that the specific gravity of the fluid is 0.8 is kinda irrelevant.

§ A block of metal weighs 500 N in air but weighs only 300 N when it is totally submerged in water. What is the specific gravity of this metal?

A. 0.4
B. 1.2
C. 1.8
D. 2.5

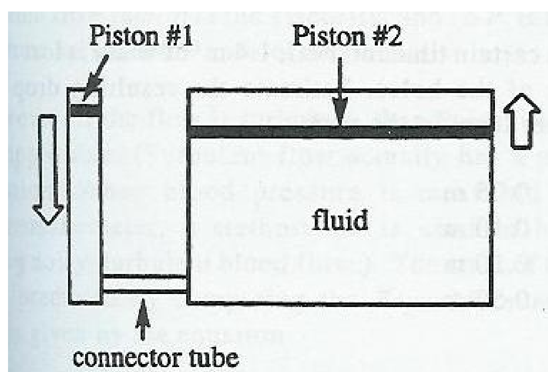
The block weighs 500 N. If it only weighs 300 N in water, then it must experience a buoyant force of 200 N. We can now find the specific gravity of the metal using the formula for buoyant force and weight (Note that the volume submerged is equal to the total volume because the metal block is "totally submerged"):

$$W = r_{object} g V_{total} = 500N$$

$$F_B = r_{water} g V_{total} = 200N$$

$$\frac{r_{object} g V_{total}}{r_{water} g V_{total}} = \frac{r_{object}}{r_{water}} = \frac{500N}{200N} = 2.5$$

§ The piston in vessel #1 has a cross-sectional area of 5 cm^2, while the piston in vessel #2 has a cross-section one-hundred times larger. What weight on the small piston will support a weight of 5000 N on the larger one?



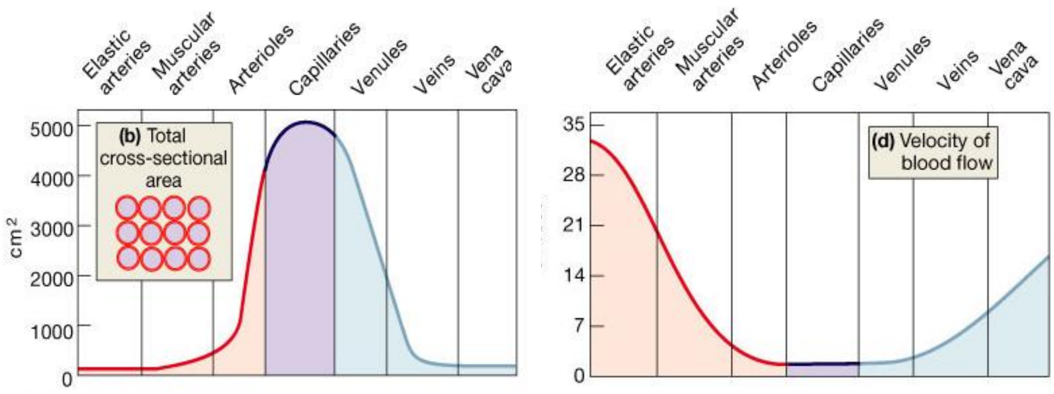
A. 50 N
B. 100 N
C. 250 N
D. 500 N

Just use the formula for Pascal's law:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1 = \frac{A_1}{A_2} F_2 = \frac{5}{500} \cdot 5000N = 50N$$

Hydrodynamics (or Fluid Dynamics)

- Hydrodynamics is the study of fluids (hydro) in motion (dynamics). There are a few big concepts to know:
  - **Continuity equation** states that the volume flow rate is equal between any 2 locations within a pipe or tube. Continuity implies conservation of mass. Don't forget that a KEY ASSUMPTION of the continuity equation is INCOMPRESSIBILITY of the fluid. This ensures that mass conservation translates into volume conservation when density is constant.
    - First, the definition of volume flow rate is: $f = Av$ where A is the cross-sectional area and v is the flow speed.
    - Next, volume flow rate is constant between 2 locations in a tube: $A_1 v_1 = A_2 v_2$
    - One physiological example is the blood flow from venules to veins. Assuming constant pressure, the flow speed of blood increases when the total cross-sectional area decreases. The only reason I used venules and veins as a good example is because pressure is relatively constant between them. If there is a difference in pressure, then that's another variable we need to account for, especially when we explain the blood speed between arteries and arterioles.

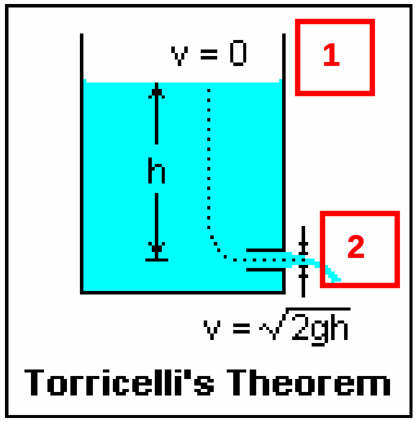


  - **Bernoulli's equation** is the energy conservation law for fluids.
    - This only applies to ideal fluids, which the MCAT will assume most of the time. Just in case, ideal fluids have the following characteristics:
      - Incompressible (density does not change)
      - Negligible viscosity (not much friction within fluid)
      - Streamline (smooth flow)
      - Steady flow
    - Formula: $P_1 + \frac{1}{2} r v_1^{2} + r g y_1 = P_2 + \frac{1}{2} r v_2^{2} + r g y_2$

§ There are many applications of Bernoulli's equation:

Toricelli's theorem tells us the efflux speed of fluid coming out from the bottom of a large tank. Let's see how Bernoulli's equation works here. We need to make the following observations:

- Both the top of the tank and the opening on the bottom are open to air—hence, they both experience atmospheric pressure.
- If the hole at the bottom of the tank is small enough, then the water level will fall at a very slow rate—
- slow enough to assume zero velocity.



Torricelli's Theorem

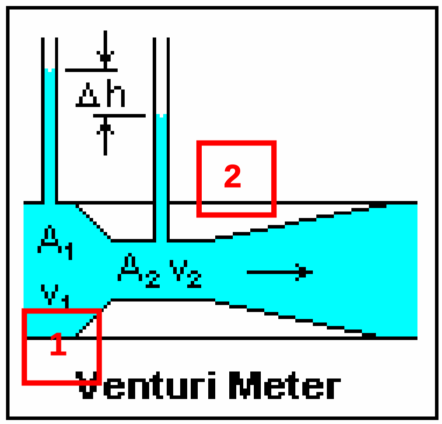
Torricelli's equation for the efflux speed of a fluid is analogous to the equation for speed of a projectile dropped from a certain height.

$$P_1 + \frac{1}{2} r v_1^{\,2} + r g y_1 = P_2 + \frac{1}{2} r v_2^{\,2} + r g y_2$$

$$P_{atm} + \frac{1}{2} r (0)^2 + r g h = P_{atm} + \frac{1}{2} r v_2^{\,2} + r g (0)$$

$$\frac{1}{2} r v_2^{\,2} = r g h \Rightarrow v_2 = \sqrt{2gh}$$

The **Venturi meter** is a good illustration of the Bernoulli theorem. We can already notice that the heights are the same because the tube is horizontal



Venturi Meter

$$P_1 + \frac{1}{2} r v_1^{\,2} + r g y_1 = P_2 + \frac{1}{2} r v_2^{\,2} + r g y_2$$

$$P_1 + \frac{1}{2} r v_1^{\,2} + r g (0) = P_2 + \frac{1}{2} r v_2^{\,2} + r g (0)$$

$$P_1 + \frac{1}{2} r v_1^{\,2} = P_2 + \frac{1}{2} r v_2^{\,2}$$

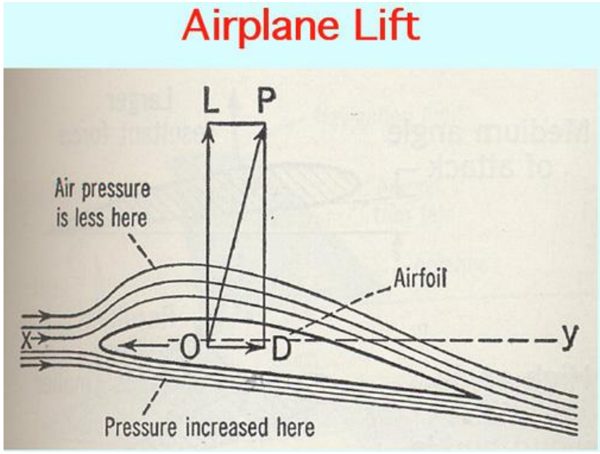
The continuity equation tells us how to relate the two velocities:

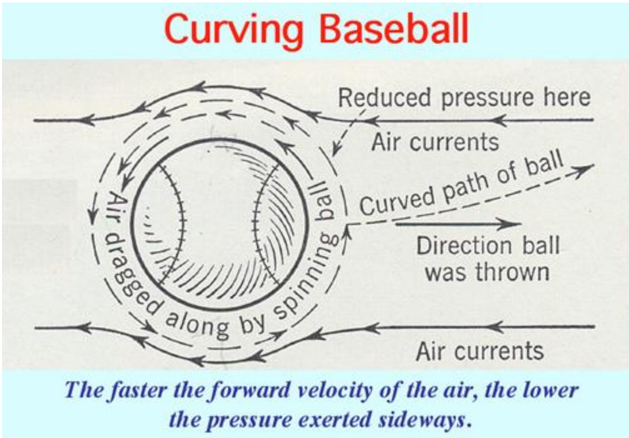
$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1}{A_2} v_1$$

$$P_1 + \frac{1}{2} r v_1^{\,2} = P_2 + \frac{1}{2} \frac{A_2^{\,2}}{A_1^{\,2}} v_1^{\,2}$$

$$P_1 = P_2 + \frac{1}{2} r \left(1 - \frac{A_2^{\,2}}{A_1^{\,2}} \right) v_1^{\,2}$$

Because A_2 is smaller than A_1, the entire term is positive. This implies that P_1 is strictly greater than P_2. We see this because the pressure at location 1 is greater (as seen in the manometer) than the pressure at location 2.





- **Bernoulli's theorem:** The faster a fluid flows, the less pressure it exerts sideways.

  From Bernoulli's theorem, we can explain why airplanes fly and why baseballs curve. Airplanes fly mainly due to the shape of their wing, which allows the air stream to travel a longer distance across the top than across the bottom. This means that the top air stream has to travel faster in order to catch up with the air stream on the bottom. Because the air is faster on top, it is also less dense and lower in pressure than the air in the bottom, creating a lift force. This same principle works for baseballs where the stitches of the ball (depending on its rotation) make the air faster in the direction of rotation by grabbing onto it. This also creates a disparity in air pressure above and below the ball, creating a force that causes the ball to curve.

Exercises:

§ If the amount of fluid flowing through a tube remains constant, how does the speed of the fluid change if the diameter of the tube increases from 2 mm to 6 mm?

A. Decrease by a factor of 9.
B. Decrease by a factor of 3.
C. Increase by a factor of 3.
D. Increase by a factor of 9.

The diameter of the tube increases by 3, which means the cross-sectional area increases by 9. According to the continuity equation, if the area increases by 9, then the flow speed decreases by 9 to preserve the volume flow rate.

§ A fluid of density r flows through a horizontal pipe with negligible viscosity. The flow is streamline with constant flow rate. If the speed at Point 1 is v and the speed at Point 2 is $v/2$, then the pressure at Point 2 is:

A. less than the pressure at Point 1 by $(3/8)rv^2$

B. less than the pressure at Point 1 by $(5/8)rv^2$

C. greater than the pressure at Point 1 by $(3/8)rv^2$

D. greater than the pressure at Point 1 by $(5/8)rv^2$

There's no real trick to this question. Just solve Bernoulli's equation. Don't forget that horizontal pipe implies zero change in elevation.

$$P_1 + \frac{1}{2}rv_1{}^2 + rgy_1 = P_2 + \frac{1}{2}rv_2{}^2 + rgy_2$$

$$P_1 + \frac{1}{2}rv_1{}^2 + rg(0) = P_2 + \frac{1}{2}rv_2{}^2 + rg(0)$$

$$P_1 + \frac{1}{2}rv_1{}^2 = P_2 + \frac{1}{2}rv_2{}^2$$

$$P_1 + \frac{1}{2}rv^2 = P_2 + \frac{1}{2}\frac{v^2}{4}$$

$$P_1 = P_2 + \frac{3}{8}rv^2$$

§ If fluid flows in such a way that the continuity equation is violated ($A_1v_1 \neq A_2v_2$), then we may conclude that:

A. the fluid is not incompressible. This assumption is **key** to continuity equation.

B. energy is not conserved. We can't violate conservation of energy.

C. the fluid has high viscosity. High viscosity will still allow for continuity equation.

D. the cross-sectional area at point 2 is actually larger than it should be. This is just a poor excuse.

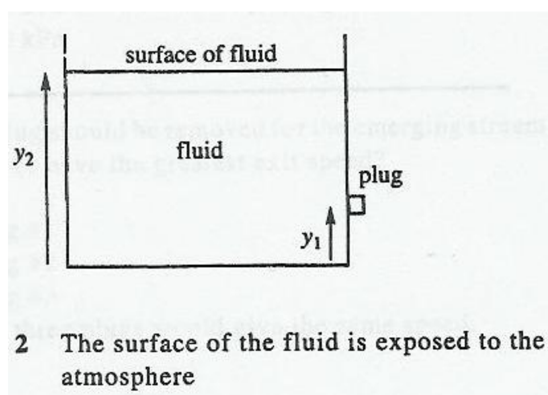
§ If the plug is pulled out, what is the speed of the water as it exits the hole? (Ignore viscosity and assume that the hole is small so that the speed at which the level of water in the tank is falling is negligible.)

A. $\sqrt{2gy_1}$

B. $\sqrt{2gy_2}$

C. $\sqrt{g(y_2 - y_1)}$

D. $\sqrt{2g(y_2 - y_1)}$



This is a direct application of Toricelli's equation (you should memorize) except that the relevant change in elevation is $(y_2 - y_1)$.