Jeffrey Seguritan
10/21/2007

Show that: $$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=-\cos 2x$$

Simplified expression before taking derivative

$$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=-\cos 2x$$

Converted cot x and tan x to their basic definitions

$$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\frac{\cos x}{\sin x}}+\frac{\cos^2 x}{1+\frac{\sin x}{\cos x}}\right)=-\cos 2x$$

Then brought both terms them under the same denominator

$$\frac{d}{dx}\left(\frac{\sin^2 x}{\frac{\sin x+\cos x}{\sin x}}+\frac{\cos^2 x}{\frac{\cos x+\sin x}{\cos x}}\right)=-\cos 2x$$

$$\frac{d}{dx}\left(\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}\right)=-\cos 2x$$

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Show that: $$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=-\cos 2x$$

Then used to quotient rule to take the derivative

$$\frac{d}{dx}\left(\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}\right)=-\cos 2x$$

$$\frac{(\sin x+\cos x)(3\sin^2 x\cos x-3\cos^2 x\sin x)-(\sin^3 x+\cos^3 x)(\cos x-\sin x)}{(\sin x+\cos x)^2}=-\cos 2x$$

$$\frac{3\sin^3 x\cos x-3\sin^2 x\cos^2 x+3\cos^2 x\sin^2 x-3\cos^3 x\sin x-\sin^3 x\cos x+\sin^4 x-\cos^4 x+\cos^3 x\sin x}{(\sin x+\cos x)^2}=-\cos 2x$$

$$\frac{2\sin^3 x\cos x-2\cos^3 x\sin x+\sin^4 x-\cos^4 x}{(\sin x+\cos x)^2}=-\cos 2x$$

Multiplied everything out

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Show that: $$\frac{d}{dx}\left(\frac{\sin^2 x}{1+\cot x}+\frac{\cos^2 x}{1+\tan x}\right)=-\cos 2x$$

Since we really can't do much more with this numerator, let's try to manipulate the other side, and see if we can get it to equal

Multiply both sides by the denominator

$$\frac{2\sin^3 x\cos x-2\cos^3 x\sin x+\sin^4 x-\cos^4 x}{(\sin x+\cos x)^2}=-\cos 2x$$

$$2\sin^3 x\cos x-2\cos^3 x\sin x+\sin^4 x-\cos^4 x=-\cos 2x(\sin x+\cos x)^2$$

$$2\sin^3 x\cos x-2\cos^3 x\sin x+\sin^4 x-\cos^4 x=-(\cos^2 x-\sin^2 x)(\sin x+\cos x)^2$$

This is a trig identity for cos 2x

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Show that: $\dfrac{d}{dx}\left(\dfrac{\sin^2 x}{1+\cot x}+\dfrac{\cos^2 x}{1+\tan x}\right)=-\cos 2x$

Now we multiply out the right-hand side and see if it equals the left-hand side

$2\sin^3 x\cos x-2\cos^3 x\sin x+\sin^4 x-\cos^4 x=-(\cos^2 x-\sin^2 x)(\sin x+\cos x)^2$

$-(\cos^2 x-\sin^2 x)(\sin x+\cos x)^2$

So does the right-hand side equal the left-hand side?

$(\sin^2 x-\cos^2 x)(\sin x+\cos x)^2$

$(\sin^2 x-\cos^2 x)(\sin^2 x+2\sin x\cos x+\cos^2 x)$

$\sin^4 x+2\sin^3 x\cos x+\cos^2 x\sin^2 x-\cos^2 x\sin^2 x-2\sin x\cos^3 x-\cos^4 x$

$\sin^4 x+2\sin^3 x\cos x-2\sin x\cos^3 x-\cos^4 x$

Yep!

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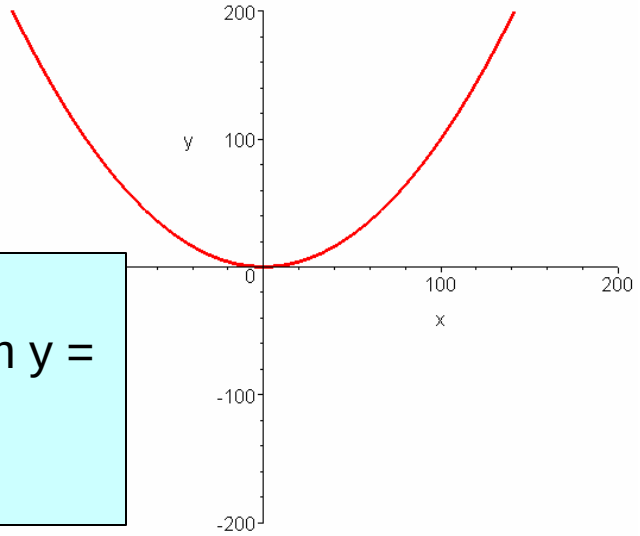
A car is traveling at night along a highway shaped like a parabola with its vertex at the origin. The car starts at a point 100m west and 100m north of the origin and travels in an easterly direction. There is a statue located 100m east and 50 m north of the origin. At what point on the highway will the car's headlights illuminate the statue?

First things first… I need to know the equation of the parabola. But I have all the information I need:

(1) Vertex at (0,0)
(2) Point at (100,100)

This tells me that the parabola is of the form $y = kx^2$ (no shifting of the parabola)

This tells me that the parabola is of the form $y = (1/100)x^2$, since (-100,100) is a point.



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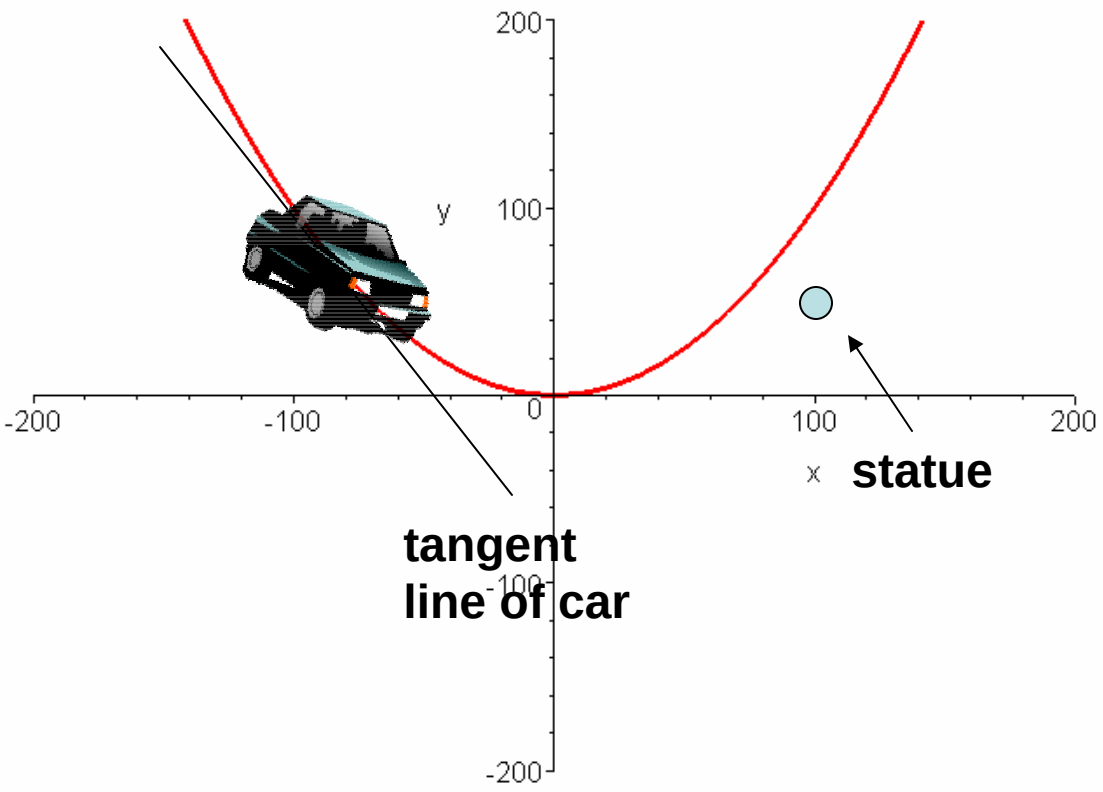
So you want to know at what point on the parabola the tangent line of the car will coincide with the location of the statue



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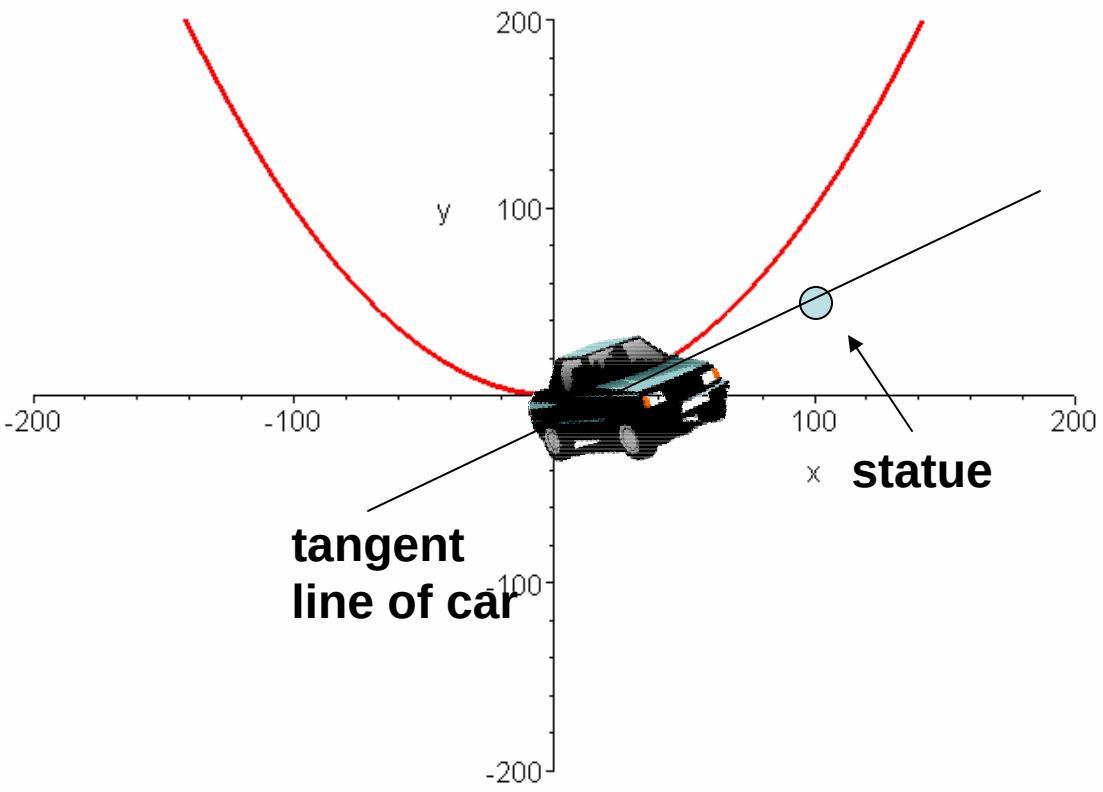

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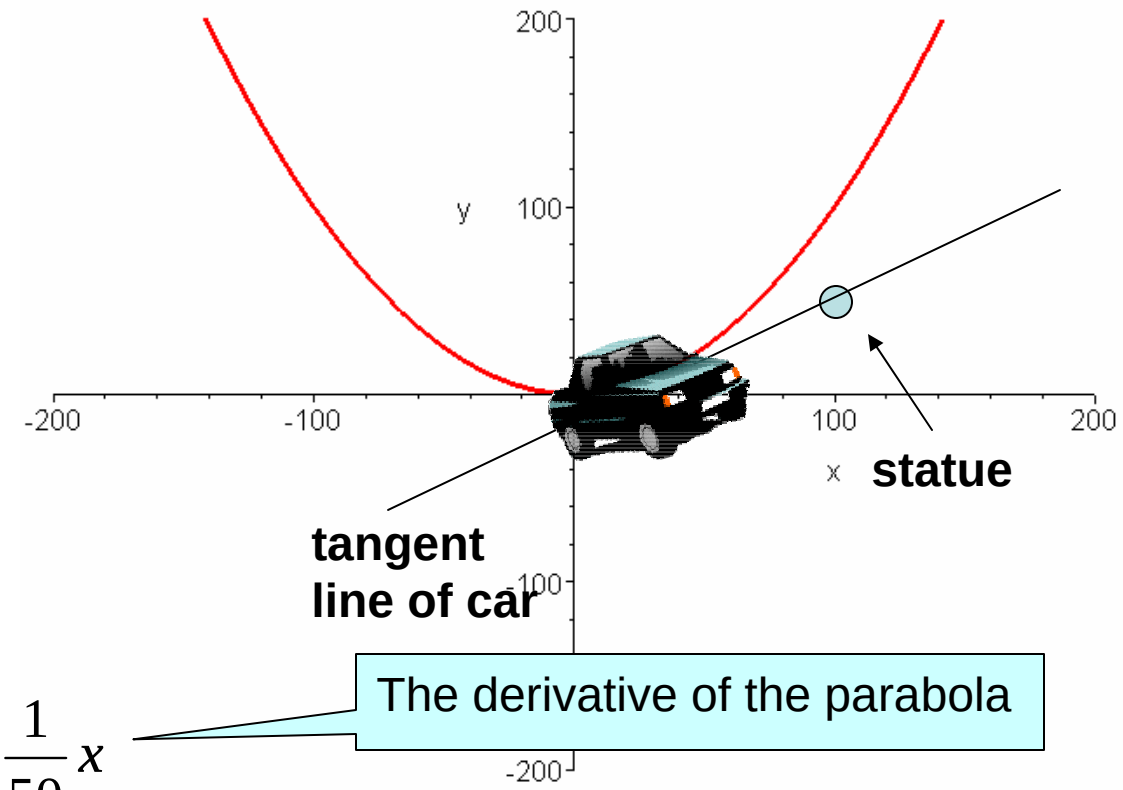
What is the equation of the tangent line to the curve?
- Slope is the derivative
- (100,50) is a point on the tangent line
- (x, x2/100) is a point on the parabola that also must be on the tangent line



$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2}{100}\right) = \frac{2x}{100} = \frac{1}{50}x$$

The derivative of the parabola

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$$\frac{y_2 - y_1}{x_2 - x_1} = slope$$

This is really y, on the parabola

$$\frac{(x^2/100) - 50}{x - 100} = \frac{1}{50}x$$

This is the slope

$$\frac{x^2}{100} - 50 = \frac{1}{50}x(x - 100)$$

$$\frac{x^2}{100} - 50 = \frac{1}{50}x^2 - 2x$$

$$\frac{x^2}{100} - 2x + 50 = 0$$

$$x = 100 \pm 50\sqrt{2}$$

Why do we get two possible x-values?

If (1/50)x is the derivative, we want to find a line in which the slope between (x,y) on the parabola and (100,50) is equal to (1/50)x

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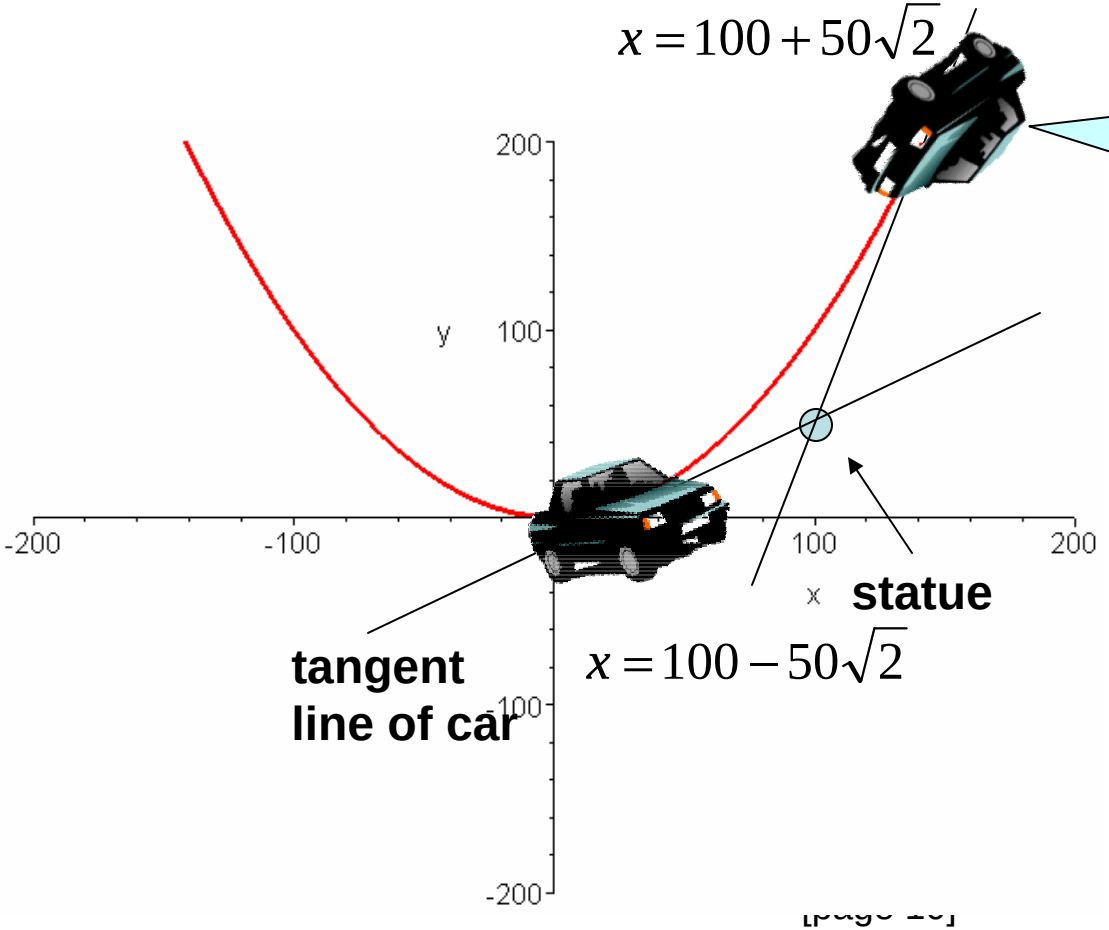

This is why we get two possible values, but as you can see, this value is meaningless, since the headlights are pointing away.

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So what are the coordinates of the point along the parabola?

$$x = 100 - 50\sqrt{2} = 29.3$$

$$y = \frac{x^2}{100} = \frac{(100 - 50\sqrt{2})^2}{100} = 8.6$$

$$(x, y) = (29.3, 8.6)$$