Brief Guide to Graphing Rational Expressions

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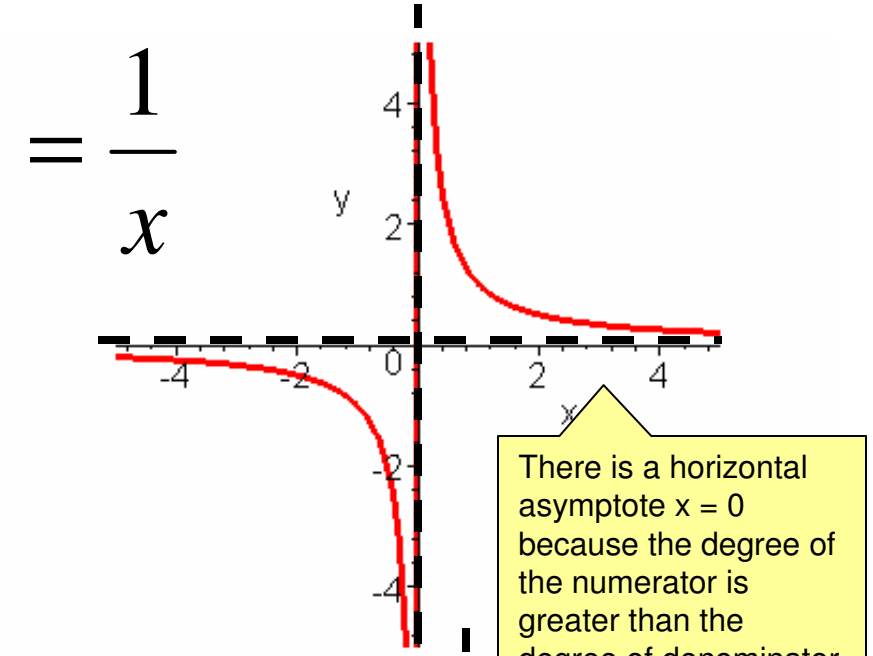
Since x = 0 is a zero of the denominator, it is a vertical asymptote. Note that when the multiplicity of the zero is ODD (here, it is 1), the curve does NOT approach the asymptote in the SAME direction.

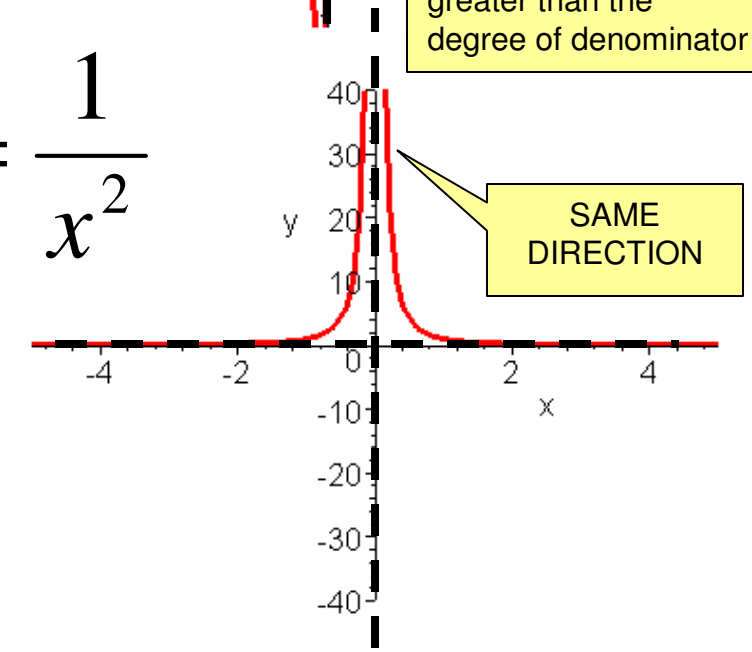
$$f(x) = \frac{1}{x}$$



There is a horizontal asymptote x = 0 because the degree of the numerator is greater than the degree of denominator

In this case, the zero of the denominator is also x = 0; hence, the vertical asymptote. The multiplicity of the zero is EVEN (here, it is 2) so the curve approaches the asymptote in the SAME direction.

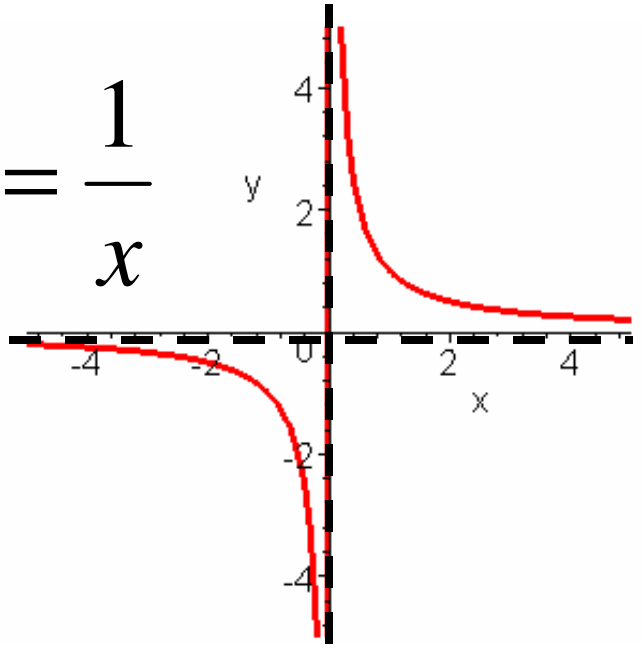
$$f(x) = \frac{1}{x^2}$$



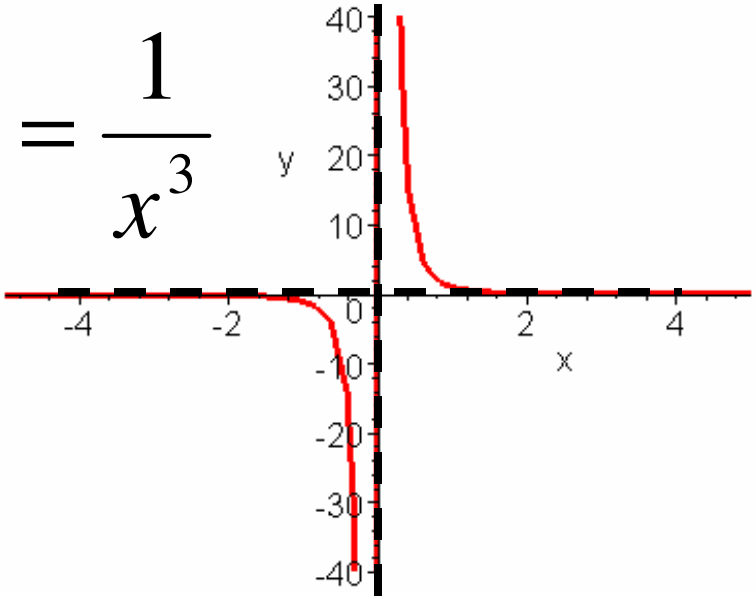
SAME DIRECTION

Note that for ODD powers in the denominator (zero power in numerator), the behavior of the curve is very similar, in that the curve ALWAYS approach the asymptote in OPPOSITE directions

$$f(x) = \frac{1}{x}$$

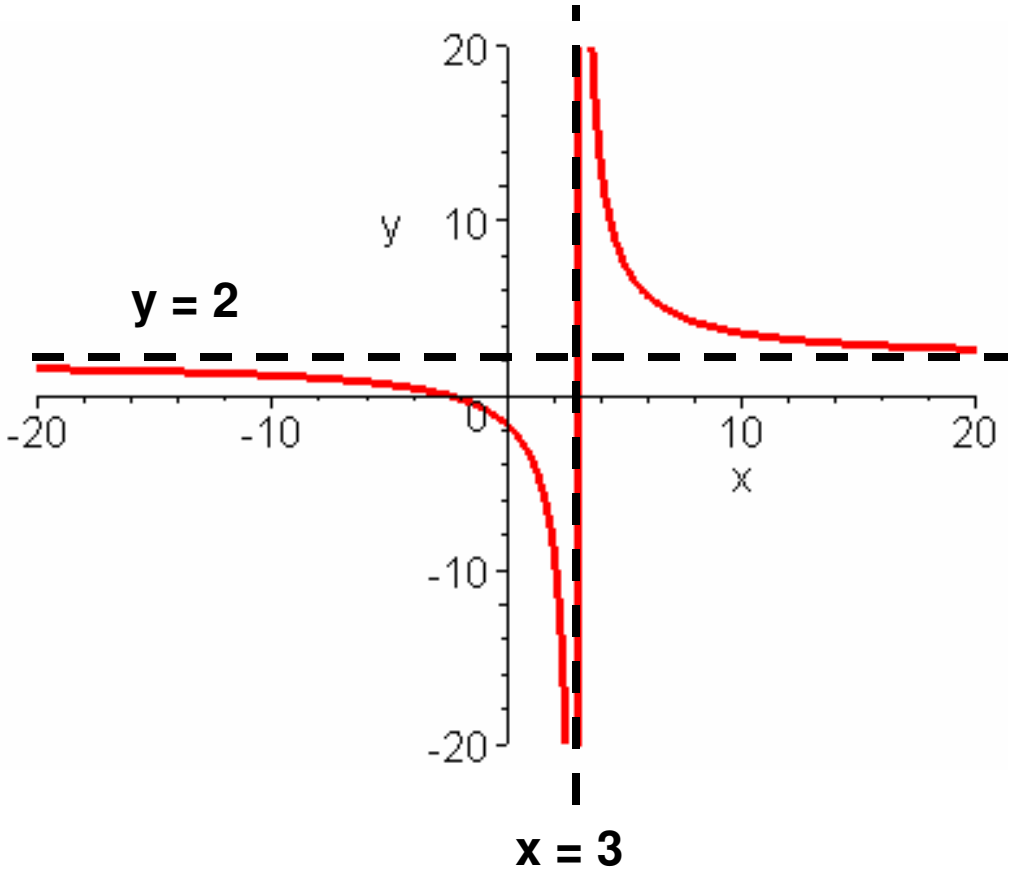


$$f(x) = \frac{1}{x^3}$$



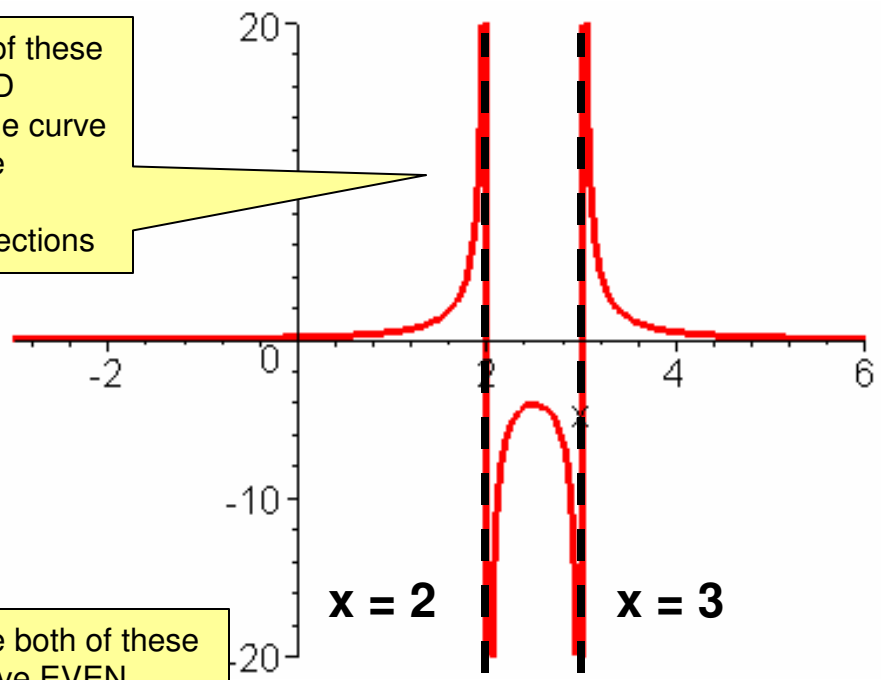
$$f(x) = \frac{2x-5}{x-3}$$

•There is a vertical asymptote at x = 3, because that is the zero of the denominator.
• There is ALSO a HORIZONTAL asymptote. In this case, it is at y = 2. This happens when the degree of the numerator EQUALS to degree of the denominator. The horizontal asymptote is simply the division of those two coefficients. So 2x divided by x is equal to 2. If the equation was (3x – 5) / (x – 3), the horizontal asymptote would be y = 3.



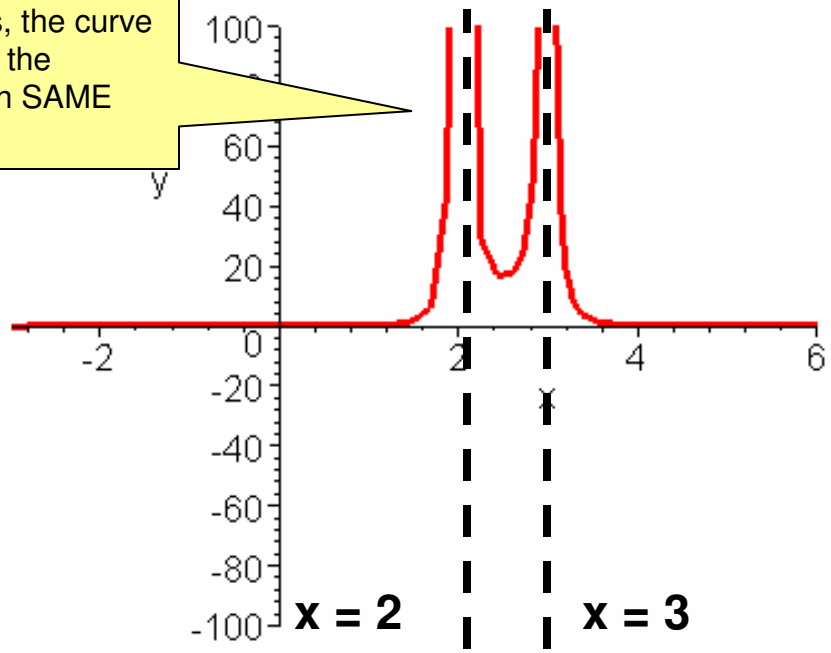
$$f(x) = \frac{1}{(x-2)(x-3)}$$

Because both of these roots have ODD multiplicities, the curve approaches the asymptote in OPPOSITE directions



$$f(x) = \frac{1}{(x-2)^2(x-3)^2}$$

Because both of these roots have EVEN multiplicities, the curve approaches the asymptote in SAME directions



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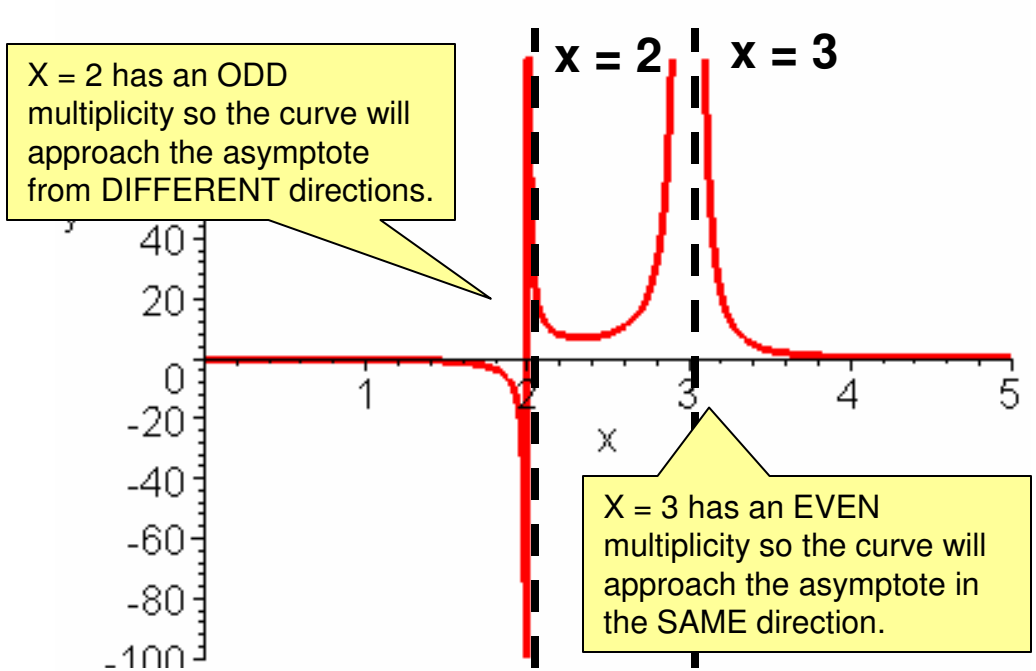
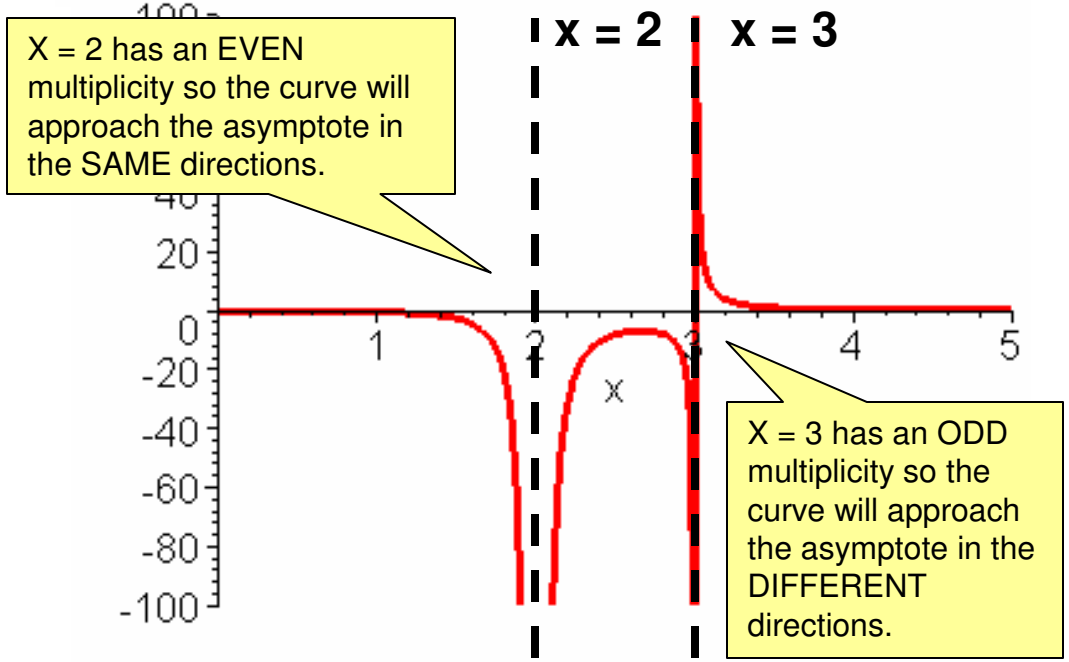


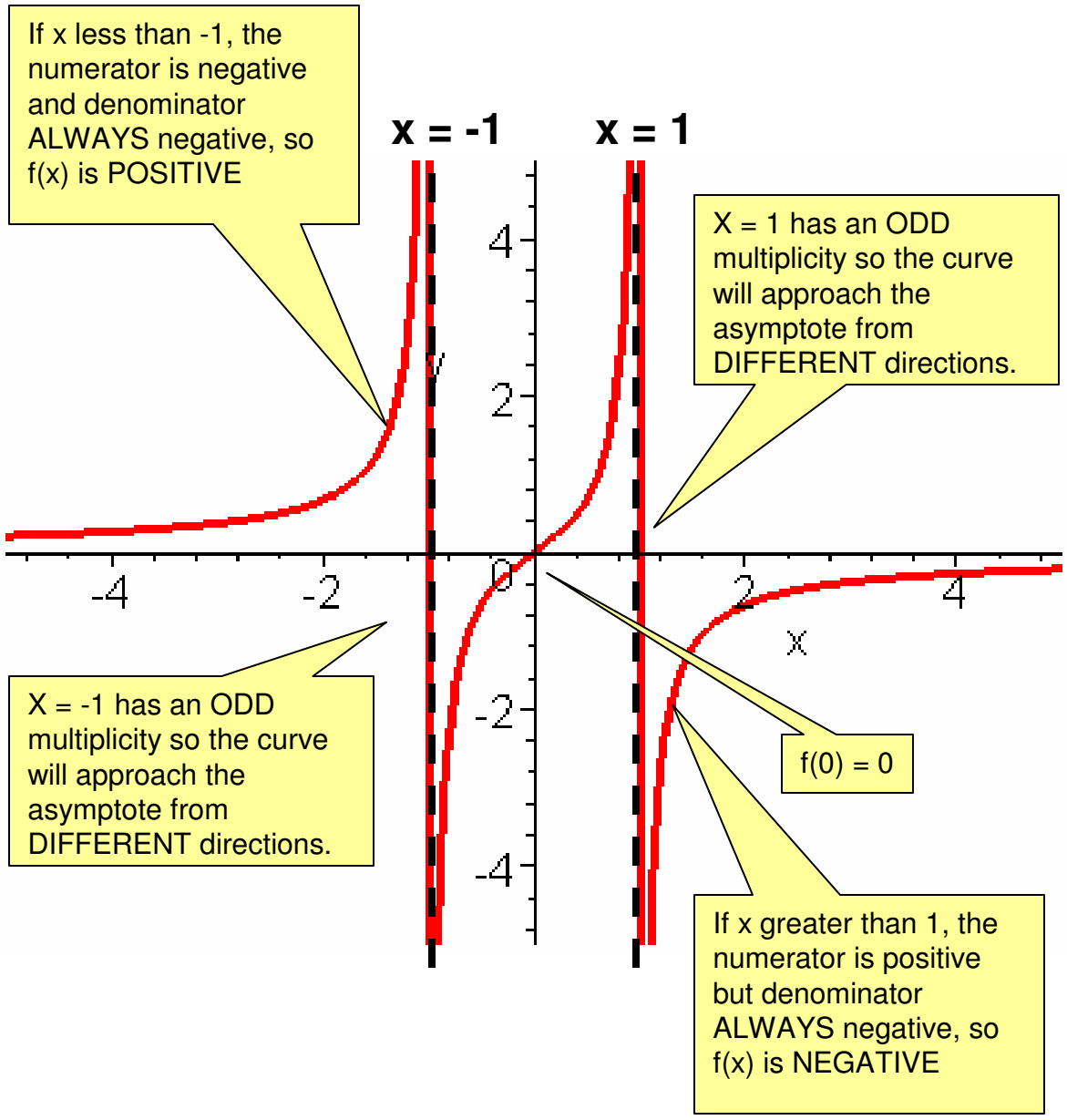
$$f(x) = \frac{1}{(x-2)^2(x-3)}$$



$$f(x) = \frac{x}{1 - x^2}$$

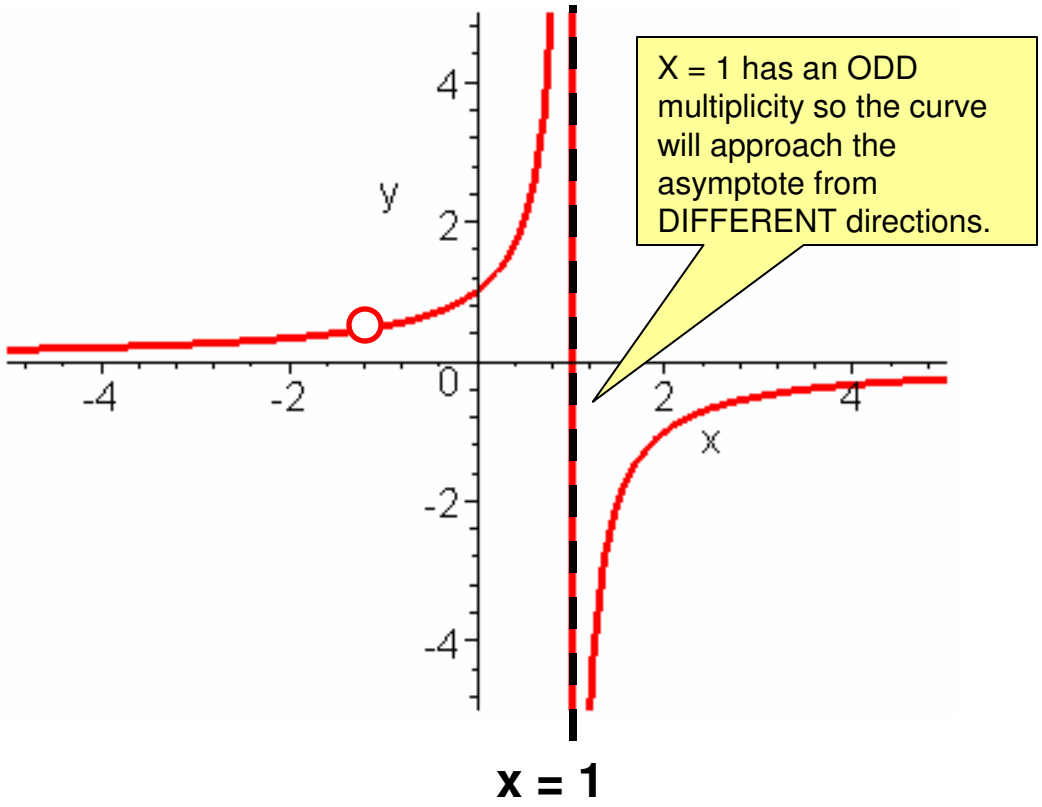
$$f(x) = \frac{x}{(1 - x)(1 + x)}$$

- Based on the denominator, there are VERTICAL asymptotes at x = 1 and x = -1
- Because the denominator has a higher degree than the numerator, there is also a HORIZONAL asymptote at y =0.



$$f(x) = \frac{x+1}{1-x^2} = \frac{\cancel{x+1}}{(1-x)\cancel{(1+x)}} = \frac{1}{1-x}$$

• Why does making the numerator x + 1 remove the asymptote at x = -1? Because since it can cancel out in the numerator AND denominator, it becomes a HOLE in the function.



$$f(x) = \frac{x^2}{x-2}$$

• There is a vertical asymptote at x = 2 because that is a zero of the denominator.
• Now note that there is a SLANT ASYMPTOTE. This happens when the degree of the numerator is ONE GREATER than the degree of the denominator. The slant asymptote depends on the ratio between the highest degree coefficient in the numerator and the highest degree coefficient in the denominator. In this case, x^2 divided by x gives a slant asymptote of y = x.